

# THEORETICAL AND EXPERIMENTAL ANALYSIS OF MICROWAVE TUNABLE RECURSIVE ACTIVE FILTERS USING POWER DIVIDERS

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## ABSTRACT

We show how power dividers can be effectively employed in the design of microwave recursive filters in strict accordance to low frequency principles. We present analytical, computer-simulated and experimental results for an active recursive band pass filter, and for a newly developed tunable recursive active filter, employing a reflection-type microwave phase shifter and implemented on a Duroïd substrate in the 2.75-3.75 GHz range.

## INTRODUCTION

Recent realizations of recursive and transversal filters have shown that a given signal flowgraph may be translated into any number of physical designs [1][2][3]. As compared with these recently published papers, we describe a new circuit topology, which we analytically demonstrate to be capable of providing a recursive filter response, with no deviation from low frequency principles [4][5]. We then show how recursive filter can simply provide tunable transfer function with the help of an arbitrary well-known phase shifter structure. We consider an active recursive filter and the corresponding tunable circuit including a reflection-type analog phase shifter. Measured S-parameters of the circuits show excellent agreement with theoretical analysis.

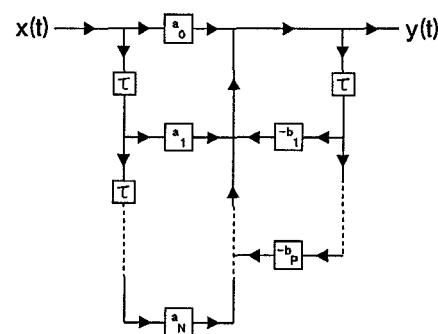
## THEORY

Recursive and transversal filters are governed by the following time-domain equation

$$y(t) = - \sum_{p=1}^P b_p y(t-p\tau) + \sum_{k=0}^N a_k x(t-k\tau)$$

where  $x(t)$  [ $y(t)$ ] is the input [output] of the system.

The general signal flowgraph of a  $(N;P)$  order recursive filter is shown in figure 1.



- Figure 1 -

Implementation of the schematic requires multiple constant delay increments  $\tau$ , amplitude weighting elements  $\{a_k\}$ ,  $\{b_p\}$ , and a mean of combining the elementary delayed signal components. In the frequency domain, the corresponding equation becomes

$$H(f) = \frac{\sum_{k=0}^N a_k e^{-2j\pi f k \tau}}{1 + \sum_{p=1}^P b_p e^{-2j\pi f p \tau}}$$

$H(f)$  is seen to be periodic with period  $f_0 = 1/\tau$ , and can be put into the form

$$H(f) = H_0(f) * \sum_{i=-\infty}^{+\infty} \delta(f - i f_0)$$

$H_0(f)$  is called the pattern of the transfer function  $H(f)$  and  $f_0$  is the width of  $H(f)$  pattern.

We now consider the Z notation of the transfer function for a  $(N;P)$  order filter.

$$H(z) = \frac{\sum_{k=0}^N a_k z^{-k}}{1 + \sum_{p=1}^P b_p z^{-p}} = T_z(h(n)) = \sum_{n=0}^{+\infty} h_n z^{-n}$$

The necessary and sufficient condition for a linear filter of response  $h(n)$  to be stable gives

$$|H(z)| \leq \sum_{n=0}^{+\infty} |h_n| |z^{-n}| < \infty \quad \text{for } |z| \geq 1$$

Stability of the recursive model requires that poles of  $H(z)$  are within the circle  $|z| \leq 1$ .

## POWER DIVIDERS

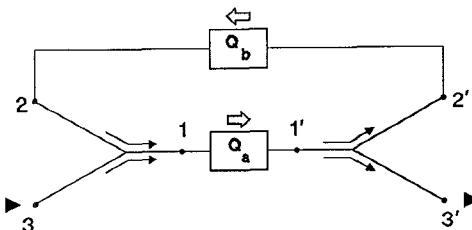
Recursive filters can turn to advantage of using power dividers. Couplers and dividers provide great flexibility in matching input and output ports, and more generally, flexibility in the filter design. This approach enables filter branches to be easily associated and then to be designed separately. Moreover, the transposition of low frequency principles to microwaves domain becomes possible, especially for periodicity, stability and tunability concepts.

## RECURSIVE FILTER SYNTHESIS

In this paragraph, we focus on a first order recursive filter governed by the following time-domain equation

$$y(t) = a_0 x(t) - b_1 y(t-\tau)$$

Recursive topology using 3-port devices is presented in figure 2.



- Figure 2 -

The S-matrix of the ideal  $50\Omega$  3-port divider/combiner is given by

$$(S) = e^{-2j\pi f\theta} \times \begin{pmatrix} 0 & \alpha & \beta \\ \alpha & 0 & 0 \\ \beta & 0 & 0 \end{pmatrix} \quad ; \quad |\alpha|^2 + |\beta|^2 = 1$$

where  $\theta$  is the delay-time introduced by the 3-port component.

For the synthesis of this structure, no hypothesis are made on  $Q_a$  except that the quadripole is matched to  $50\Omega$  to all its ports.  $Q_b$  is assumed to be a  $l_b$  length of  $50\Omega$  transmission line. Then, theoretical analysis shows that both input and output ports of the global filter are matched to  $50\Omega$  ( $S_{11} = S_{22} = 0$ ), and that transmission parameter  $S_{21}(f)$  can be written into the form

$$S_{21}(f) = \frac{-\beta^2 S_{a21} e^{-4j\pi f\theta}}{1 - \alpha^2 S_{a21} e^{-2j\pi f(2\theta + \tau_b)}} \quad (1)$$

where  $\tau_b$  is the delay-time introduced by the  $l_b$  line length. We now set that the 3-port devices are 3 dB power dividers ( $|\alpha|^2 = |\beta|^2 = 1/2$ ). From equation (1), we find

$$|S_{21}(f)| = \frac{|S_{a21}|/2 e^{j\phi(f)}}{1 - |S_{a21}|/2 e^{-2j\pi f\tau}}$$

$$\text{where } \tau = 2\theta + \tau_a + \tau_b = 1/f_0$$

$\tau_a$  is the delay-time introduced by  $Q_a$

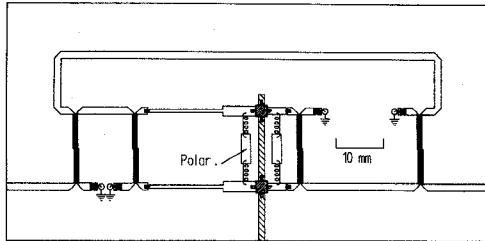
$\phi(f)$  is a linear frequency dependent function.

$$\text{We set } |S_{21}|_{\text{Max}} / |S_{21}|_{\text{Min}} = \frac{1 + |S_{a21}|/2}{1 - |S_{a21}|/2} = 10 \text{ (20dB)}$$

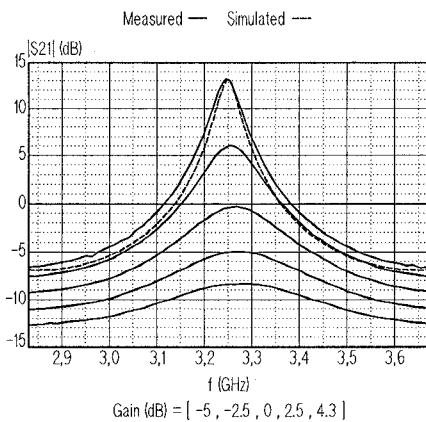
$$\text{We find } |S_{a21}| = 1.64 \text{ (4.3 dB)} \text{ and } a_0 = -b_1 = |S_{a21}|/2 = 0.82.$$

The filter is designed in the 2.8-3.7 GHz band, with  $f_c = 3.25$  GHz the center frequency and  $f_0 = 0.9$  GHz the width of  $H(f)$  pattern. The quadripole  $Q_a$  is a balanced amplifier using two 3 dB six-strip Lange couplers and two bipolar transistors. Flat gain in the specified frequency band has been reached with the help of two cascaded microstrip transmission lines.

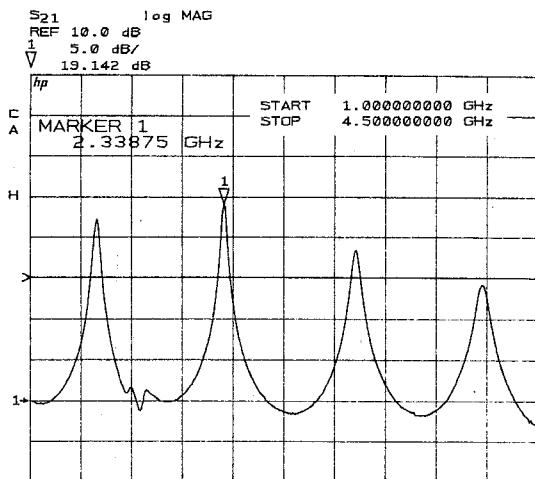
Corresponding active recursive filter circuit, including the amplifier, and four six-strip 3 dB Lange couplers is presented in figure 3. Good agreement is illustrated between computer-simulated and experimental results in figure 4. Measured bandwidth of the filter is 60 MHz around 3.248 GHz (1.8% bandwidth). Figure 5 illustrates the pseudo-periodical aspect of  $S_{21}(f)$  in the 1.0-4.5 GHz band. According to low frequency principles, the filter is near instability at frequency 2.33 GHz, due to higher gain of the amplifier at the bottom of the band.



- Figure 3 -



- Figure 4 -



- Figure 5 -

## TUNABLE RECURSIVE FILTER SYNTHESIS

In this paragraph, we show how recursive filters can simply provide tunable transfer functions. We start from the expression of a transversal transfer function. To tune the filter means to electrically modify the circuit characteristics in order to shift the transfer function  $H(f)$  over a frequency band noted  $\Delta f$ .

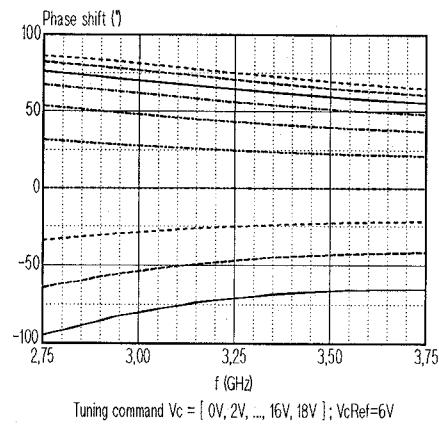
The resulting frequency response can be put into the following form

$$H(f - \Delta f) = \sum_{k=0}^N a_k e^{-2j\pi k \tau (f - \Delta f)} = \sum_{k=0}^N A_k e^{-2j\pi k \tau}$$

These results can readily be extended to the recursive model. In this case, we find

$$A_k = a_k e^{jk\varphi} ; B_p = b_p e^{jp\varphi} \text{ with } \varphi = 2\pi \Delta f \tau$$

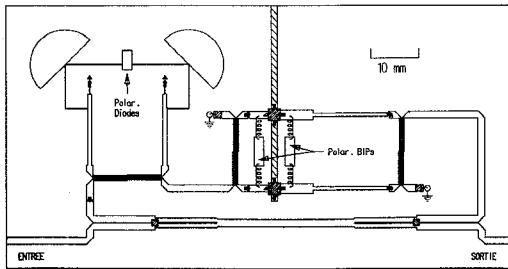
Physically, to tune the filter means to add a supplementary phase shift to each weighting parameter  $a_k$  or  $b_p$ . This phase shift must be frequency independent and proportional to the expected transfer function frequency shift. That may be easily implemented by an arbitrary well-known phase shifter structure. We built a reflection-type analog phase shifter [6] comprising hyperabrupt doping profile varactor diodes with a 9:1 capacitance ratio and a Lange coupler in the 2.75-3.75 GHz band. The circuit on each of the direct and coupled ports of the coupler are constituted with one varactor diode and two transmission line lengths. We use a  $180^\circ$  circular stub to supply the diodes bias voltage. As shown in figure 6, the relative flatness of the  $\pm 75^\circ$  phase shift, for various tuning command states over the working band validate our approach.



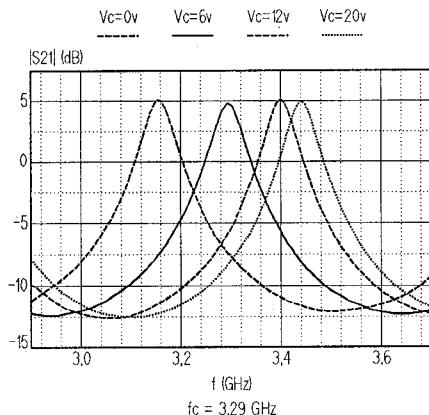
- Figure 6 -

We now directly insert the phase shifter into the feedback branch of the previous active structure. The corresponding tunable active filter circuit, including the phase shifter, is shown in figure 7. Couple of 2-section Wilkinson dividers now associate the filter branches. Indeed, the contribution of the longitudinal dimension of these components to the parameter  $\tau$ , significantly reduces the size of the final circuit.

Figure 8 shows measured S-parameters for the filter in the 2.75-3.75 GHz band for multiple tuning command values. Amplifier gain has been adjusted, for each value of the tuning command voltage, to compensate for the phase shifter losses. Tuning frequency range is 300 MHz wide, approximately 10% around the center frequency  $f_c = 3.29$  GHz. Figure 9 underlines the pseudo-periodical aspect of the transfer function  $S_{21}(f)$  in the 1.5-6.0 GHz range and validate our theoretical and experimental approach. Once again, the filter is near instability at the bottom of the frequency band, due to the amplifier gain shape.



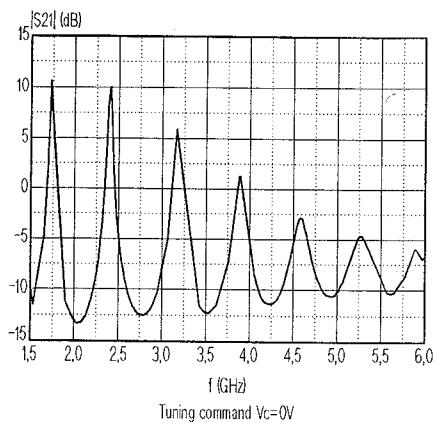
- Figure 7 -



- Figure 8 -

## CONCLUSION

The main purpose of this paper has been to analyse the methodology of using power dividers in tunable recursive filter design, in keeping with the strict interpretations of low frequency principles. The design approach has been outlined for an active recursive circuit employing a balanced amplifier, and designed in the 2.75-3.75 GHz band. The last step in the filter design has been to directly insert a classical reflection-type analog phase shifter structure into the recursive loop, to help in the realization of a tunable recursive response.



- Figure 9 -

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